Exercise 54

For the following exercises, sketch a graph of the quadratic function and give the vertex, axis of symmetry, and intercepts.

$$f(x) = x^2 - 6x - 1$$

Solution

In order to more easily graph the quadratic function, write it in vertex form by completing the square. The following algebraic identity is necessary.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -6$, which means $B = -3$ and $B^2 = 9$. Add and subtract 9 from the right side and use the identity.

$$\begin{aligned} f(x) &= (x^2 - 6x + 9) - 1 - 9 \\ &= (x + (-3))^2 - 10 \\ &= (x - 3)^2 - 10 \end{aligned}$$

Therefore, the vertex is $(3, -10)$, and the axis of symmetry is $x = 3$. To determine the y-intercept, set $x = 0$.

$$f(0) = (0 - 3)^2 - 10 = (9) - 10 = -1$$

Therefore, the y-intercept is $(0, -1)$. To get the x-intercept, set $y = 0$ and solve the equation for x.

$$\begin{aligned} 0 &= (x - 3)^2 - 10 \\ 10 &= (x - 3)^2 \end{aligned}$$

Take the square root of both sides.

$$\sqrt{10} = \sqrt{(x - 3)^2}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - 3$.

$$|x - 3| = \sqrt{10}$$

Remove the absolute value sign by placing $\pm$ on the opposite side.

$$x - 3 = \pm\sqrt{10}$$

Add 3 to both sides.

$$x = 3 \pm \sqrt{10}$$

This means $x = \{3 - \sqrt{10}, 3 + \sqrt{10}\}$, and the x-intercepts are $(3 - \sqrt{10}, 0)$ and $(3 + \sqrt{10}, 0)$.

A graph of the function is shown below.